Exchange-Based Diffusion in Hb-Graphs: Highlighting Complex Relationships

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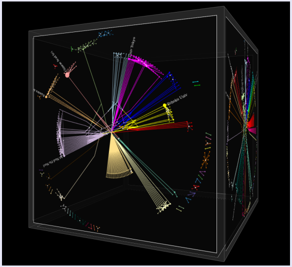
Xavier Ouvrard, Jean-Marie Le Goff, Stéphane Marchand-Maillet

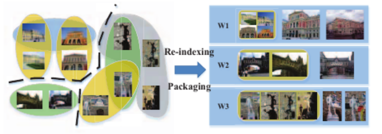
Introduction

Information space exploration

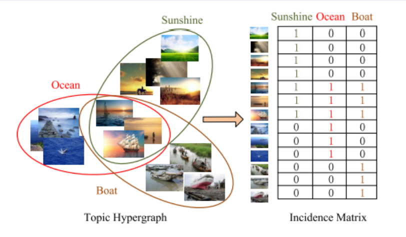


DataHyperCube: prototype in Ouvrard et al. [2018a]

Multimedia indexing



Generation of similar image sets Xu et al. [2016] ©IEEE 2016



Topic hypergraph Zhu et al. [2017] © IEEE 2017

In this paper

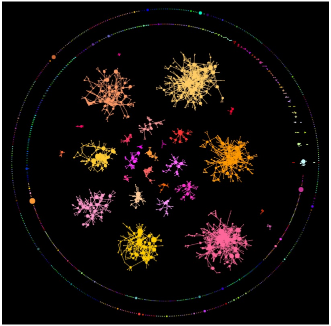


Figure 1: Search: 3D Graph: Organisations

Problem statement

How to **highlight** important information in a co-occurence network ?

RQ

Can we find a network model and a diffusion process that:

- rank vertices
- and rank set / multiset of vertices

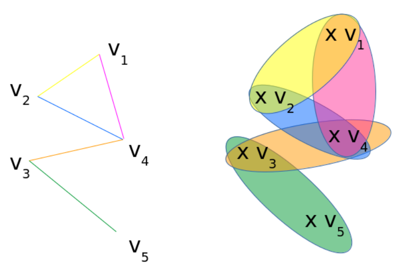
Contribution

A theoretical framework that allows easy handling of weights on vertices => family of multisets => **hb-graphs**
Intermediate step: a **diffusion process** that supports the retrieval of information:

- not only on **vertices**
- but also on **hb-edges.**

Hypergraphs

From graphs to hypergraphs



- **Hypergraphs** $\equiv$ generalisation of graphs to multiple nodes' links
- Hypergraphs introduced by Berge and Minieka [1973].

Definition

Bretto [2013]:

- **Hypergraph** $\mathcal{H}$: a family of subsets of a vertex set
- **Hyperedges**: elements of the family

Two visions

- **set of elements** of power set of nodes $\rightsquigarrow$ set view
- **extension of graphs** $\rightsquigarrow$ n-adic relationship view

Diffusion in hypergraphs

RW with weighted hyperedges

Zhou et al. [2007], Ducournau and Bretto [2014]:

- p(e_k chosen) $\propto$ weight
- p(vertex in e_k) uniform
- $p(v_i, v_j) = \sum_{k=1}^{p} w_e(e_k) \frac{h_{ik}}{d_i} \times \frac{h_{jk}}{\delta_k}$
- stationary state => $\pi(v_i) \propto d(v_i)$.

RW with weighted vertices

Bellaachia and Al-Dhelaan [2013]:

- hyperedge based vector of weights on vertices
- $p(v_i$ chosen in $e_k) \propto w_{e_k}(v_i)$
- application to conference proceedings => ranking of words by the random walk

An application to Multimedia Indexing

Xu et al. [2016]: **multi-feature indexing of images** to help to retrieval

- Each image associated to n most similar images => hyperedges
- Hyperedges weighted by average similarity
- Hypergraph separated into k sub-hypergraphs by spectral approach
- RW on each subhypergraph => significant images => build a revert index

Multisets

Multiset and operations

- **Multiset**: a universe and a multiplicity function $A_m = (A, m)$
- **Natural multiset**: the range of the multiplicity function is a subset of $\mathbb{N}$.
- In natural multisets: two views:
  - weighted set
  - collection of objects
- **Support** of the multiset: elements of the universe that have non zero multiplicity
- **m-cardinality** of a multiset A_m: sum of all multiplicity of elements of A.
- Operations on multisets exist:
  - **Inclusion** allows definition of **submset**
  - Union, intersection and sum
- More in Singh et al. [2007].

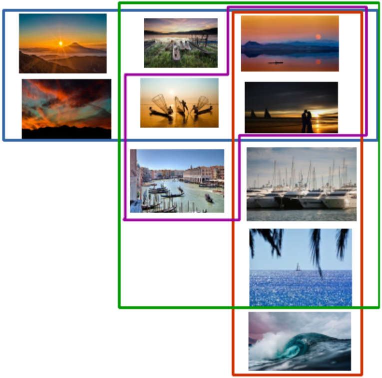
Photo from https://www.pexels.com/photo/sailboats-racing-163318/

- A = {boat, ocean, sunset, land}
- $m\,(\text{boat}) = 6$
- $m(\text{ocean}) = 1$
- $m(\text{sunset}) = 0$
- $m\,(\text{land}) = 1$

Hb-graphs: extending hypergraphs

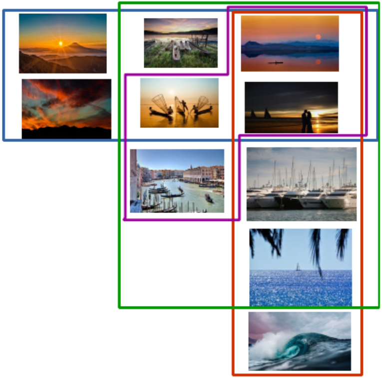
Hyper-Bag-graph or hb-graph

- **Hb-graph**: family of multisets - called **hb-edges** - with:
  - same universe V, called **vertex set**.
  - support a subset of V.
  - each hb-edge has its own multiplicity function.
- **Natural hb-graph**: when all multiplicity functions have their range included in $\mathbb{N}$
- **Support hypergraph**: hypergraph of the support of the multisets
- **Star of a vertex**: multiset of all hb-edges where the vertex is, with a multiplicity the vertex multiplicity in this hyperedge
- **m-degree of a vertex**: m-cardinality of the star of this vertex
- **hypergraph**: natural hb-graph with multiplicity function ranges in $\{0, 1\}$



Photos from https://www.pexels.com/photo/sailboats-racing-163318/

Hb-graphs: extending hypergraphs



Photos from https://www.pexels.com/photo/sailboats-racing-163318/

	sunset	boat	ocean	person
	1	0	0	0
	1	0	0	0
	1	4	0	0
	1	3	0	3
	1	1	1	1
	1	2	1	2
	0	28	0	17
	0	1	1	0
	0	0	1	0

Diffusion by exchange in hb-graphs

Principle

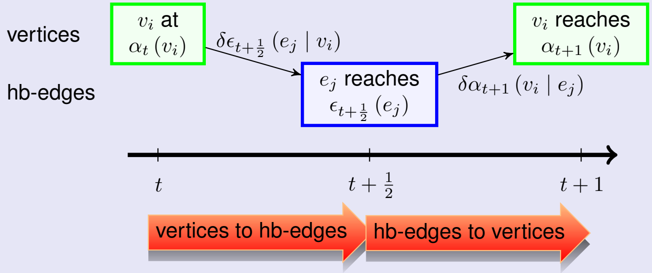


Figure 2: Diffusion by exchange: principle Ouvrard et al. [2018b] © IEEE 2018

$$\delta\epsilon_{t+\frac{1}{2}}(e_j \mid v_i) = \frac{m_j(v_i)\, w(e_j)}{d_{w,m}(v_i)} \alpha_t(v_i).$$

$$\delta\alpha_{t+1}(v_i \mid e_j) = \frac{m_j(v_i)\, w(e_j)}{\#_m e_j} \epsilon_{t+\frac{1}{2}}(e_j).$$

Results and evaluation

Methodology

- 2 parts in experimentation:
  - generation of random hb-graphs => validation of the approach
  - comparison to a classical random walk
- **Visualisation** of the hb-graphs generated: highlight hb-edges and vertices
- **Eccentricity** in a graph: $e(v_0) = \max_{v \in V} \min l\,(\text{path}\,(v, v_0))$

  => extension to hb-graph straightforward:
  - requires strict path in a hb-graph
  - link between strict path and path in the support hypergraph
- Evaluation using **relative eccentricity** between vertices in $S \subset V$ and $V \backslash S$; if vertices are not connected eccentricity is at $-\infty$
- S built using a threshold s_V on α value of vertices in V:
  - above: in S
  - below: in $V \backslash S$

Visualisation

Visualisation of exchange-based diffusion

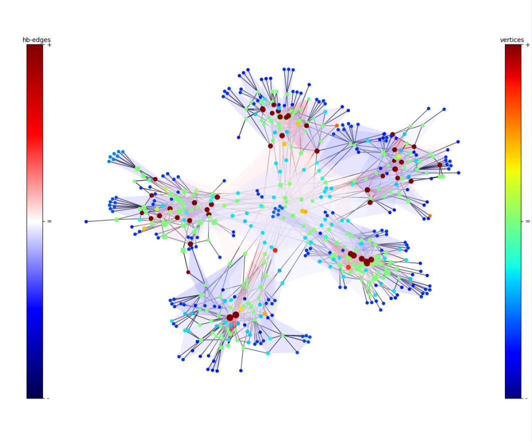


Figure 3: From Ouvrard et al. [2018b] © IEEE 2018

On example

- 548 vertices
- 300 hb-edges
- 5 groups
- 10 vertices in between the 5 groups

Results I

Diffusion by exchange

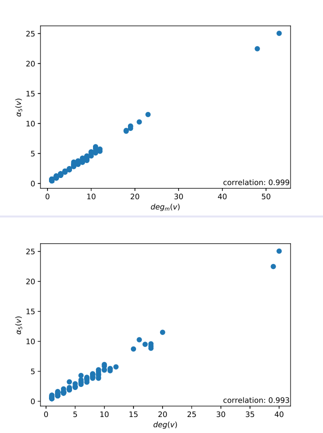


Figure 4: Alpha value of vertices at step 5 and (m-)degree of vertices. Ouvrard et al. [2018b] © IEEE 2018

Random walks

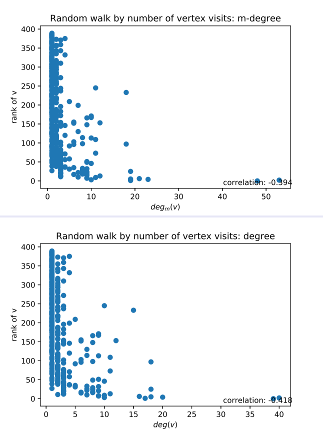


Figure 5: Rank obtained by 100 RW after vertex total discovery and (m-)degree of vertices Ouvrard et al. [2018b] © IEEE 2018

Results II

Diffusion by exchange: Epsilon value of hb-edge at stage 4.5

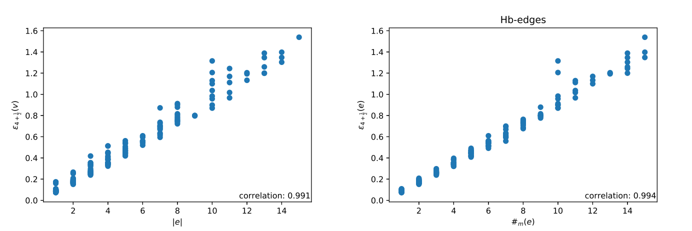


Figure 6: Epsilon value of hb-edge at stage $4+\frac{1}{2}$ and (m-)cardinality of hb-edge. Ouvrard et al. [2018b] © IEEE 2018

- 5 iterations in diffusion exchange: 0.009 s
- 100 RW with total covering: take 6.31 s

Future work and Conclusion

Conclusion

- Possibility of using hb-graphs for highlighting networks
- Intermediate step of the process allows additional information

Future work

- Applications to develop:
  - multimedia indexing
  - information retrieval
- Strong basis to refine the approach of Xu et al. [2016]

Thank you

Questions?

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